

Thermodynamical description of modified generalized Chaplygin gas model of dark energy

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We consider a universe filled by a modified generalized Chaplygin gas together with a pressureless dark matter component. We get a thermodynamical interpretation for the modified generalized Chaplygin gas confined to the apparent horizon of FRW universe, while dark sectors do not interact with each other. Thereinafter, by taking into account a mutual interaction between the dark sectors of the cosmos, we find a thermodynamical interpretation for interacting modified generalized Chaplygin gas. Additionally, probable relation between the thermal fluctuations of the system and the assumed mutual interaction is investigated. Finally, we show that if one wants to solve the coincidence problem by using this mutual interaction, then the coupling constants of the interaction will be constrained. The corresponding constraint is also addressed. Moreover, the thermodynamic interpretation of using either a generalized Chaplygin gas or a Chaplygin gas to describe dark energy is also addressed throughout the paper.

I. INTRODUCTION

Recent observations imply an expanding universe while its rate of expansion is extremely increased [1–4]. A primary model introduced to explain this kind of expansion uses Cosmological Constant (CC) in order to model the source of this expansion [5]. In fact, Standard cosmology, including a universe filled by a CC along with the cold dark matter (CDM) and baryonic matters, is in line with the observations and standard particle physics theory [5–10]. In addition, this model (Λ CDM) is satisfying the generalized second law of thermodynamics in its current stage of expansion indicating that the universe maintains this phase [11]. However, the unknown nature of dominated fluid or CC along with the coincidence and fine tuning problems are some of the unsatisfactory parts of this model [5].

Moreover, CC is classified into a more general class of fluids named dark energy (DE) using to describe the current phase of expansion [12]. There are also numerous models introducing a dynamic DE [13–43]. Additionally, It seems that the DE models in which state parameter is not constant have a better fitting with observations compared with CC [44–49]. One of the models which includes non-constant state parameter is Chaplygin gas which has interesting features [12, 50]. In this model, the pressure of DE is related to its density as

$$p_D = -\frac{B}{\rho_D}, \quad (1)$$

where B is a constant and subscript D is used to indicate that it is a model for DE. This model attracted more investigators to itself [50–59]. In addition, the generalization of Chaplygin gas (GCG) is

$$p_D = -\frac{B}{\rho_D^\alpha}, \quad (2)$$

while $0 \leq \alpha \leq 1$ and covers CG for $\alpha = 1$ [60]. On one hand, this model can be reinterpreted as an entangled mixture of DE and DM [61], while on the other hand, this dual role is eliminated by observations [61, 62]. Finally, we should note that one can consider GCG as a candidate for dynamical DE [63, 64]. It is also useful to mention that this model preserves the thermodynamical equilibrium conditions [11]. Moreover, this generalization can be modified (MGCG) as

$$p_D = A\rho_D - \frac{B}{\rho_D^\alpha}, \quad (3)$$

while we have again $0 \leq \alpha \leq 1$, whenever A is a constant [56, 60]. Additionally, the pressure profiles of GCG and CG are obtainable by substituting $A = 0$ and $A = 0$ together with $\alpha = 1$ respectively. It is easy to show that

$$\rho_D = \left(\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right)^{\frac{1}{1+\alpha}}, \quad (4)$$

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as the density profile of MGCG while C is an integration constant and $A \neq -1$. MGCG also covers the radiation density profile for $A = \frac{1}{3}$. Moreover, for $A = 0$, it behaves as a pressureless matter in early universe and blurs a cosmological constant-like behavior in the last stage of the universe [56, 60]. For the state parameter $\omega_D = \frac{p_D}{\rho_D}$ we get

$$\omega_D = A - B \frac{a^{3(1+A)(1+\alpha)}(1+A)}{Ba^{3(1+A)(1+\alpha)} + C(A+1)}, \quad (5)$$

meaning that the MGCG model looks like a mixture of a perfect fluid with $\omega_{pf} = A$ and a GCG with $\omega_{GCG} = B \frac{a^{3(1+A)(1+\alpha)}(1+A)}{Ba^{3(1+A)(1+\alpha)} + C(A+1)}$ which is in line with Eq. (3). We should note again that the density profile and the state parameter of considering GCG are obtainable by inserting $A = 0$ in Eqs. (4) and (5) respectively. The result of considering CG are also covered by substituting $A = 0$ together with $\alpha = 1$ in Eqs. (4) and (5). Additionally, $\omega_D \rightarrow -1$ in the long run limit ($a \gg 1$) when $A > -1$, and the state parameter of phantom regime is also accessible in this limit for $A < -1$. Various aspects of MGCG, as a dynamical model for DE, was also investigated [65–71]. Meanwhile, some thermodynamical aspects of the Chaplygin gas models, including CG, GCG and MGCG, can also be found in Refs. [72–81]. The state parameter, heat capacity in constant volume and thermodynamic behavior of MGCG in terms of temperature and volume are studied in [79]. The mutual correspondence between MGCG and DBI-essence as well as the validity of generalized second law of thermodynamics in a FRW universe filled by a MGCG are shown in ref. [80]. Moreover, it is also shown that the generalized second law of thermodynamics may be valid on Hubble, particle and apparent horizon of the FRW universe filled by a MGCG together with a non-interacting magnetic field [81]. A comprehensive review on the DE and modified gravity which is also including the Chaplygin gas models can be found in [12].

From observational point of view, the dark sectors of universe, including DE and DM, can interact with each other [12, 82–90]. Additionally, such interactions may solve the coincidence problem [90–98], if they lead to decay DE into DM [97, 98]. These results are also in line with the observations supporting the idea of using a dynamical model to describe DE [44–49]. Such likely mutual interactions between the Chaplygin gas models and DM were also investigated [61, 91, 97, 99–107]. Thermodynamics of the GCG model interacting with CDM was studied which shows that the second law of thermodynamics can be valid in this scenario [108]. Moreover, it seems that thermal fluctuations may correct the entropy of gravitational systems [109]. These corrections are not limited to the gravitational system and indeed, they are available in all thermodynamical systems [109]. In the cosmological setups, thermal fluctuations of system may be attributed to the mutual interaction between the dark sectors [98, 107, 110–113]. Therefore, it is useful to study the thermodynamics of mutual interaction between the dark sectors of cosmos helping us to get a more actual model for describing DE.

Our goal in this paper is finding a thermodynamical interpretation for the mutual interaction between MGCG, as a dynamical model for DE, and DM by using thermal fluctuations approach [98, 107, 110–113]. For this propose, we take into account that our universe, described by the FRW metric

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (6)$$

includes a MGCG and a pressureless DM component. For the FRW metric, $a(t)$ is the scale factor while $k = -1, 0, 1$ is the curvature parameter corresponding to open, flat and closed universes respectively [5]. Because apparent horizon of dynamical spacetimes can be considered as a causal boundary [114–116], we use the corresponding horizon for the FRW metric located at [117, 118]

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a(t)^2}}}. \quad (7)$$

Finally, since WMAP data indicates a flat universe, we set $k = 0$ in our study [5].

The paper is organized as follows. In the next section, we consider the FRW universe filled by a MGCG together with a pressureless DM while they do not interact with each other. Thereinafter, we find a relation for the entropy changes of MGCG. In section III, the mutual interaction between the dark sectors is taken into account. We will find thermodynamical description for the interacting MGCG model. Throughout this paper, we address the results of considering either GCG or CG instead of MGCG by using their relation to MGCG. Section IV is devoted for a summary and concluding remarks. For the sake of simplicity, we take $G = \hbar = c = 1$ throughout this paper.

II. THERMODYNAMICAL DESCRIPTION OF NON-INTERACTING MODIFIED GENERALIZATION OF CHAPLYGIN GAS

Friedmann equations for the FRW universe filled by a DE together with a pressureless DM are

$$H^2 = \frac{8\pi}{3}(\rho_m + \rho_D), \quad (8)$$

and

$$-2\frac{\ddot{a}}{a} - H^2 = 8\pi p_D, \quad (9)$$

where dot denotes derivative with respect to time, and $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter. Moreover, ρ_i and p_D are the density of i^{th} fluid and the pressure of DE respectively. If we define $\rho_c \equiv 3\frac{H^2}{8\pi}$ and use Eqs. (8), we get

$$1 = \Omega_D + \Omega_m, \quad (10)$$

where $\Omega_i = \frac{\rho_i}{\rho_c}$ is the fractional energy density of the i^{th} component of the dark sector. Energy-momentum conservation law implies

$$\dot{\rho}_m + \dot{\rho}_D + 3H(\rho_m + p_m + \rho_D + p_D) = 0. \quad (11)$$

where dot denotes again derivative with respect to time. The energy-momentum conservation law can be decomposed into

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (12)$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0, \quad (13)$$

when the dark sectors do not interact with each other. For MGCG confined to the apparent horizon of the flat FRW universe, the Gibb's law implies

$$TdS_D = dE_D + p_D dV. \quad (14)$$

In this equation, S_D is the entropy of MGCG while $V = \frac{4\pi}{3}\tilde{r}_A^3$ and $E_D = \rho_D V$ are the volume of the flat FRW universe and the energy of MGCG respectively. Since thermodynamic equilibrium condition implies that T (the temperature of MGCG) should be equal to that of the apparent horizon [118–120], we obtain

$$T = \frac{H}{2\pi} = \frac{1}{2\pi\tilde{r}_A}, \quad (15)$$

where we have used Eq. (7) for the flat FRW universe ($k = 0$). Because

$$dV = 4\pi(\tilde{r}_A)^2 d\tilde{r}_A = -4\pi H^{-4} dH, \quad (16)$$

and

$$dE_D = \rho_D dV + V d\rho_D, \quad (17)$$

we get

$$dS_D = \frac{2\pi}{H}((\rho_D + p_D)dV + V d\rho), \quad (18)$$

leading to

$$dS_D^0 = \frac{2\pi}{H_0^3} \left(1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}\right) \left(\frac{1}{(1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}) + u_0} - \frac{3\Omega_D^0}{2}\right) dH_0 \quad (19)$$

for the entropy changes of the MGCG model. The subscript/superscript (0) is used to emphasize the non-interacting parameters, and we have also defined $u \equiv \frac{\dot{\Omega}_D}{\Omega_D} = \frac{\dot{\rho}_D}{\rho_D}$ for simplicity. In deriving the above equation, we used

$$d\rho_D = \frac{\dot{\rho}_D}{\dot{H}} dH, \quad (20)$$

where $\dot{\rho}_D$ is evaluated by using Eq. (13). In addition, one should take derivative with respect to time from (8) and uses Eqs. (3), (9) and (11) to obtain the Raychaudhuri equation

$$\dot{H}_0 = -4\pi\rho_D^0((1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}) + u_0). \quad (21)$$

One may use (15) to get

$$dS_D^0 = \frac{1}{2\pi T_0^3} (1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}) (\frac{1}{(1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}) + \frac{1-\Omega_D^0}{\Omega_D^0}} - \frac{3\Omega_D^0}{2}) dT_0, \quad (22)$$

where we have used $u_0 = \frac{1-\Omega_D^0}{\Omega_D^0}$ in getting this equation. We should note again that the subscript/superscript (0) indicates that the calculations are done for the case in which the dark sectors do not interact with each other. It is also useful to mention that one can reach the results of considering the GCG and perfect fluid models by inserting $A = 0$ and $B = 0$ in the above relations respectively. Moreover, by inserting $A = 0$ and $\alpha = 1$ we will find the changes of the CG entropy. Briefly, we get a thermodynamical description for the perfect fluid and CG models confined to the apparent horizon, when the DE candidate does not interact with the pressureless DM.

III. THERMODYNAMIC DESCRIPTION OF INTERACTING MGCG

For the case in which MGCG interacts with the pressureless DM, decomposition of the energy-momentum conservation is read as

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (23)$$

and

$$\dot{\rho}_D + 3H\rho_D(1 + \omega_D) = -Q, \quad (24)$$

indicating that the energy will be transferred to DM from MGCG for $Q > 0$. Indeed, in order to solve the coincidence problem, DE should slowly decay into DM. Therefore, the mutual interaction should be very weak in accordance with the long life of the universe. Such interaction terms have been discussed from phenomenological point of view [99, 100]. The general interaction term is written as $Q = 3H(c_1\rho_m + c_2\rho_D)$ introduced in ref. [101] for the first time, where c_1 and c_2 are the coupling constants. We should note that this interaction term is the general form of those introduced in refs. [101, 104–107], and attracted more attempts to itself [12, 61, 102, 103]. Use Eqs. (8), (3), (23) and (24) to get the Raychaudhuri equation as

$$\dot{H} = -4\pi\rho_D((1 + A - \frac{B}{(\rho_D)^{\alpha+1}}) + u). \quad (25)$$

Bearing the definition of u in mind and use (10) to get

$$d\rho_D = \frac{\dot{\rho}_D}{\dot{H}} dH = \frac{3H(\rho_D + p_D) + Q}{4\pi\rho_D((1 + A - \frac{B}{(\rho_D)^{\alpha+1}}) + \frac{1-\Omega_D}{\Omega_D})}, \quad (26)$$

where we have used (24) and (3) to obtain the last equation. Finally, by following the recipe of previous section, we get

$$\frac{dS_D}{dH} = \frac{2\pi}{H^3} (\frac{3H\rho_D(1 + A - \frac{B}{(\rho_D)^{\alpha+1}}) + Q}{3H\rho_D((1 + A - \frac{B}{(\rho_D)^{\alpha+1}}) + \frac{1-\Omega_D}{\Omega_D})} - \frac{3}{2}\Omega_D), \quad (27)$$

for the entropy changes of the interacting MGCG model. Indeed, this mutual interaction affects the entropy changes of MGCG studied in the previous section. It is argued that the horizon entropy has a logarithmic correction which is

due to the thermal fluctuations [109]. This modification of entropy is available in all thermodynamical systems, and it is not limited to the gravitational systems [109]. In cosmological setups, it seems that the mutual interaction between the dark sectors of the cosmos may induce weak thermal fluctuations to the cosmos contents leading to correct the entropy [98, 107, 110–113]. In order to find a relation between thermal fluctuations and the dark sectors mutual interaction, we express the entropy of the interacting MGCG model as [98, 107, 109–113]

$$S_D = S_D^0 + S_D^1 + S_D^2. \quad (28)$$

In the above equation, S_D^0 and $S_D^1 = -\frac{1}{2} \ln CT_0^2$ are the entropy of MGCG, when there is no mutual interaction between the dark sectors of the universe, and logarithmic correction to the entropy, which is due to the thermal fluctuations, respectively. S_D^2 also concerns higher order terms which are so weak in the gravitational and cosmological systems [98, 109]. Moreover, $C = T_0 \frac{dS_D^0}{dT_0}$ is the dimensionless heat capacity, and it is also useful to mention again that this analysis is valid for all thermodynamical systems [109]. It is a matter of calculation to show that

$$C = T_0 \frac{dS_D^0}{dT_0} = \frac{1}{2\pi T_0^2} \left(1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}\right) \left(\frac{1}{(1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}) + \frac{1-\Omega_D^0}{\Omega_D^0}} - \frac{3\Omega_D^0}{2}\right), \quad (29)$$

where we have used (22) to derive the above equation. Combination Eqs. (13), (20) and (21) leads to

$$\frac{d\rho_D^0}{dH_0} = \frac{3H_0(1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}})}{4\pi((1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}) + \frac{1-\Omega_D^0}{\Omega_D^0})} \quad (30)$$

Therefore, we get

$$\frac{dS_D^1}{dH_0} = -\frac{1}{2} \left[\frac{(1 + A + \alpha \frac{B}{(\rho_D^0)^{\alpha+1}})}{(\rho_D^0 + A\rho_D^0 - \frac{B}{(\rho_D^0)^\alpha})} \frac{d\rho_D^0}{dH_0} + \frac{8\frac{\pi}{H^3} + \frac{-\frac{3H}{4\pi} + \frac{d\rho_D^0}{dH_0}(A + \alpha \frac{B}{(\rho_D^0)^{\alpha+1}})}{(A\rho_D^0 - \frac{B}{(\rho_D^0)^\alpha} + \frac{3H^2}{8\pi})^2}}{\frac{1}{A\rho_D^0 - \frac{B}{(\rho_D^0)^\alpha} + \frac{3H^2}{8\pi}} - \frac{4\pi}{H^2}} \right], \quad (31)$$

where we have used

$$\frac{d}{dH_0} = \frac{d\rho_D^0}{dH_0} \frac{d}{d\rho_D^0}, \quad (32)$$

whiles $\frac{d\rho_D^0}{dH_0}$ can be found in Eq. (30). From Eq. (28), we get

$$\frac{dS_D}{dH} = \left(\frac{dS_D^0}{dH_0} + \frac{dS_D^1}{dH_0} + \frac{dS_D^2}{dH_0}\right) \frac{dH_0}{dH}, \quad (33)$$

leading to

$$\frac{dS_D^2}{dH_0} = \frac{dS_D}{dH} \frac{dH}{dH_0} - \frac{dS_D^0}{dH_0} - \frac{dS_D^1}{dH_0}, \quad (34)$$

whenever

$$\frac{dH}{dH_0} = \frac{\dot{H}}{\dot{H}_0} = \frac{\rho_D((1 + A - \frac{B}{(\rho_D)^{\alpha+1}}) + \frac{1-\Omega_D}{\Omega_D})}{\rho_D^0((1 + A - \frac{B}{(\rho_D^0)^{\alpha+1}}) + \frac{1-\Omega_D^0}{\Omega_D^0})}. \quad (35)$$

In deriving the last equation, we used Eqs. (21) and (25). Eq. (34) gives us an expression for the higher order terms of the thermal fluctuations (S_D^2) which was omitted in the previous works [98, 107, 109–113]. Moreover, we should note that since the S_D^0 and S_D^1 terms have the major contributions in the expression (36), one can withdraw the S_D^2 contribution, and writes [98, 107, 109–113]

$$\frac{dS_D}{dH} = \left(\frac{dS_D^0}{dH_0} + \frac{dS_D^1}{dH_0}\right) \frac{dH_0}{dH}. \quad (36)$$

Indeed, this relation helps us to find an expression between the mutual interaction of the dark sectors and the thermal fluctuations of the universe, up to the first order. Therefore, we provided a thermodynamical interpretation for the mutual interaction between the dark sectors of the universe. In order to solve the coincidence problem, DE should decay into DM meaning that the mutual interaction between the dark sectors must meet the $Q > 0$ condition [98]. By applying this condition to a general interaction $Q = 3H(c_1\rho_m + c_2\rho_D)$ and using Eq. (10) we get

$$\frac{c_1}{c_2} > -\frac{\Omega_D}{1 - \Omega_D} = -\frac{1}{u}, \quad (37)$$

where we have used this fact that $H\rho_c > 0$. Therefore, when the coupling constants (c_1 and c_2) meet this condition, the mutual interaction between the dark sectors may solve the coincidence problem. Additionally, our surveys help us to find the corresponding relation between the thermal fluctuations of the system and mutual interaction between the dark sectors. Finally, it is useful to note that by substituting $A = 0$ along with $\alpha = 0$ and $A = 0$ in the above formulas one can find the results of considering the interacting CG and GCG models respectively.

IV. SUMMARY AND CONCLUDING REMARKS

Since the recent observations admit an interaction between the dark sectors of the cosmos [84–90], the hopes to resolve the coincidence problem, by using this interaction, are increased [90–98]. In addition, it seems that such interactions induce thermal fluctuations into the system [98, 107, 110–113], and therefore one should be able to find a mutual relation between the thermal fluctuations and such interactions. Moreover, there are observation evidences indicating that dynamical DE models have a better fitting with observations [44–49]. All of these data motivate us to study the thermodynamics of mutual interaction between MGCG and DM. Here, we have considered MGCG as a dynamical model for DE and studied thermodynamics of the non-interacting MGCG model. Therefore, we got its corresponding thermodynamical description. In continue, we have focused on the interacting MGCG model and studied its thermodynamics. We saw that the entropy of the interacting MGCG model differs from the non-interacting case. Additionally, bearing the thermal fluctuations theory along with the Raychaudhuri equation in mind, we could find an expression for the fluctuations due to the mutual interaction between the dark sectors. The relation of our results to those of the GCG and CG models was also addressed by pointing to the corresponding limits. Finally, we showed that the solving of the coincidence problem conditions that the coupling constants of the mutual interaction between the dark sectors are not independent of each other. Indeed, they should satisfy relation (37). Briefly, our studies show that one can get an expression for the mutual interaction between the dark sectors by investigating the thermal fluctuations of the cosmos components. Loosely speaking, We could find a thermodynamical interpretation for the Chaplygin gas models, including the CG, GCG and MGCG models, and their probable interaction with the pressureless DM. The evolution of these fluctuations due to the universe expansion have a great importance, because it can help us to estimate the future of the cosmos. This could be an interesting problem for future works.

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